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CALCULATED NATURAL VERTICAL HULL FREQUENCIES AND  
NORMAL MODES OF SURVEYING SHIP AGS 26

by

John T. Cummings



STRUCTURAL MECHANICS LABORATORY  
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## ABSTRACT

The normal mode shapes, natural frequencies, and bending moment distribution of vertical flexural vibration of the hull were calculated for the AGS 26, a Surveying Ship. Parameters used in calculations with a digital computer are tabulated. The two-noded vertical natural frequency obtained with the computer agrees closely with that calculated using the Burrill empirical formula.

## INTRODUCTION

The AGS 26 is a Surveying Ship, the first ship of its class. The principal dimensions are given in Table 1. The Bureau of Ships requested that the David Taylor Model Basin calculate the normal hull modes and natural frequencies.<sup>1</sup> The first six vertical flexural modes and natural frequencies are presented here.

## PARAMETERS USED IN CALCULATIONS

The parameters for the calculation of the normal modes and natural frequencies were computed by M. Rosenblatt and Son, Inc., according to the procedures developed at the Model Basin.<sup>2</sup>

The vertical flexural response was calculated by assuming the hull to behave like a free-free continuous nonuniform beam. The mathematical formulation and solution of these equations are discussed in detail in References 3 and 4.

Since the parameters in these equations cannot be easily expressed as continuous functions of position along the ship, a numerical method of solution using finite difference equations was used. The hull was divided into twenty equally spaced sections of 14.0 ft. each. The result obtained depends on the values for ship mass, virtual mass, bending stiffness ( $EI$ ), and shear rigidity ( $KAG$ ) at each station.

Figure 1 is an outboard profile of AGS 26 showing the station locations. Figure 2 is a plot of the moments of inertia distribution with respect to the horizontal axis through the centroids of the sections.

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<sup>1</sup>References are listed on page 14.

Table 2 presents the mass and stiffness data used.

## RESULTS OF CALCULATIONS

Figure 3 shows the calculated normal mode profiles and natural frequencies of vertical vibration for the two-through seven-noded vibrations. The amplitudes illustrated are relative to a unit displacement input at the stern of the ship, Station 0. The moment distribution relative to this unit displacement (1 ft) is plotted in Figure 4 for each mode shape.

Figures 5 and 6 show the calculated response curves for Hull Stations 2, 10, and 20. A lumped damping constant as developed in Reference 5 and used in References 6 and 7 was applied; it is governed by the following equation:

$$\frac{c}{\mu\omega} = \frac{\frac{C}{\Delta x}}{\frac{m}{\Delta x} \omega}$$

where C is the lumped damping constant of section  $\Delta x$  in ton-sec/ft,

c is the distributed damping constant in ton-sec/ft,<sup>2</sup>

m is the lumped mass of section  $\Delta x$  in ton-sec<sup>2</sup>/ft,

$\mu$  is the mass per unit length in ton-sec<sup>2</sup>/ft,<sup>2</sup>

$\Delta x$  is the selected span length in ft, and

$\omega$  is the forcing frequency on the beam in radians/sec.

Values of  $c = 0.01 \mu\omega$  and  $c = 0.03 \mu\omega$  were used because they are considered appropriate values for calculations involving vertical vibration.

Response curves are plotted for the forcing function applied at Station 1. This forcing function was a unit sinusoidal force (1-ton single amplitude) constant over the frequency range of 1 to 50 cps.

Table 3 summarizes the natural frequencies of vertical flexural vibration. Table 4 shows the computation of the two-noded natural frequency using the empirical Burrill method.<sup>5,8</sup> The dimensions used in these calculations were obtained from References 9 and 10.

#### ACKNOWLEDGMENT

The programming of the computed parameters for the 7090 digital computer was carried out by Dr. E. H. Cuthill and Mrs. N. G. Millett of the Applied Mathematics Laboratory of the Taylor Model Basin.

TABLE 1  
Principal Dimensions of AGS 26

Ship Length between Perpendiculars, ft	283
Beam, ft	48
Draft, ft	15
Height to Main Deck, ft	23.5
Displacement, tons	2595

**TABLE 2**  
**Mass and Stiffness Data for Computing Vertical Flexural Vibration**

Station	ship mass $\mu \Delta x$ ton-sec <sup>2</sup> /ft	$\frac{\Delta x}{KAG} \times 10^5$ <sup>*</sup> 1/ton-ft	$\frac{\Delta x}{EI} \times 10^8$ ft./ton	
0			0	$\Delta x = 14.00$ ft. $E = 13.4 \times 10^3$ ton/in. <sup>2</sup> $G = 5.36 \times 10^3$ ton/in. <sup>2</sup>
1/2	1.92	22.727		
1		3.506	15.329	
1 1/2	2.51	2.693		
2		2.728	9.652	<p><sup>*</sup> Note: Moments of inertia for full stations were calculated by M. Rosenblatt &amp; Son, Inc. Those for half stations were taken from the moment curve prepared by Rosenblatt. The Applied Mathematics Laboratory, DTMB, used the full station data in its calculations.</p>
2 1/2	4.56	2.764		
3		2.735	9.223	
3 1/2	6.07	2.612		
4		2.407	9.184	
4 1/2	7.96	2.073		
5		1.741	8.957	
5 1/2	11.19	1.505		
6		1.329	8.724	
6 1/2	10.75	1.187		
7		1.080	8.552	
7 1/2	12.59	0.991		
8		0.916	8.452	
8 1/2	13.35	0.854		
9		0.811	8.502	
9 1/2	13.15	0.783		
10		0.800	8.436	
10 1/2	13.84	0.757		
11		0.704	7.429	
11 1/2	11.85	0.656		
12		0.645	7.304	
12 1/2	11.41	0.653		
13		0.665	7.429	
13 1/2	9.99	0.679		
14		0.700	7.207	
14 1/2	8.61	0.724		
15		0.754	6.636	
15 1/2	6.99	0.794		
16		0.846	6.046	
16 1/2	5.49	0.913		
17		1.000	5.947	
17 1/2	3.46	1.102		
18		1.211	5.915	
18 1/2	2.32	1.296		
19		1.811	6.327	
19 1/2	1.55	5.224		
20			0	



**TABLE 3**  
**Natural Frequencies of Vertical Flexural Vibration**

Mode	Number of Nodes	Vertical Flexural Frequency cps
1st	2	2.98
2nd	3	5.93
3rd	4	9.05
4th	5	12.30
5th	6	15.59
6th	7	18.31

**TABLE 4**  
**Calculation of Two-Noded Vertical Natural Frequency of AGS 26 by Burrill Empirical Formula**

1.	Beam = B = 48 ft Draft = d = 15 ft Height to Main Deck = D = 23.5 ft	Displacement = Δ = 2595 tons Length between perpendiculars = L = 283 ft
2.	$N = \frac{\beta}{\sqrt{1 + \frac{B}{2d}} \sqrt{1 + r}} \sqrt{\frac{BD^3}{\Delta L^3}} \quad r = \frac{3.5D^2(3a^3 + 9a^2 + 6a + 1.2)}{L^2(3a + 1)}$ $a = \frac{B}{D}$	
3.	$a = \frac{B}{D} = \frac{48}{23.5} = 2.04$ $r = \frac{3.5(23.5)^2 (3 \times 2.04^3 + 9 \times 2.04^2 + 6 \times 2.04 + 1.2)}{283^2 (3 \times 2.04 + 1)}$ $r = 0.259$	
4.	β = 97,500	
5.	$N = \frac{97,500}{\sqrt{1 + \frac{48}{30}} \sqrt{1 + 0.259}} \sqrt{\frac{48 \times 23.5^3}{2595 \times 283^3}}$ $= 175.68 \text{ cpm}$ $N = 2.93 \text{ cps}$	
6.	2.93 cps compares favorably with 2.98 cps, the frequency of the two-noded mode computed by digital methods.	

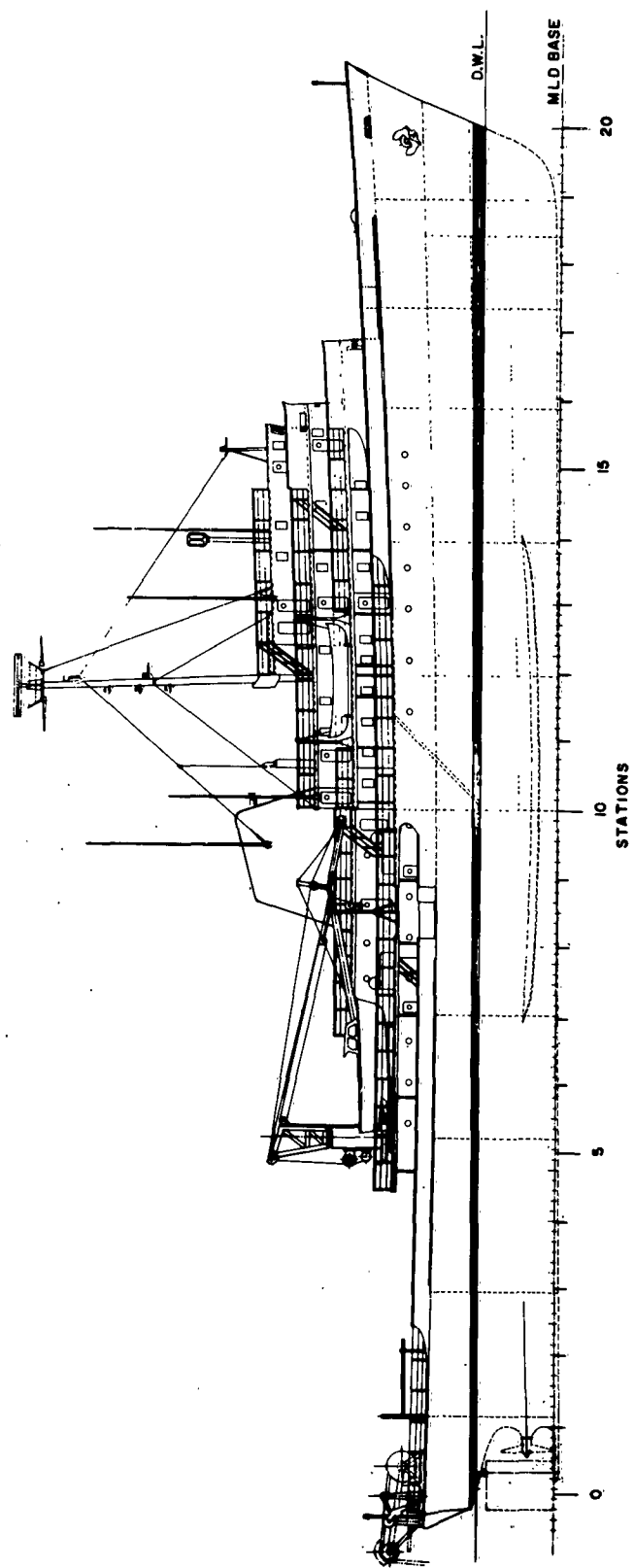


Figure 1 - Outboard Profile of AGS 26 Showing Station Locations

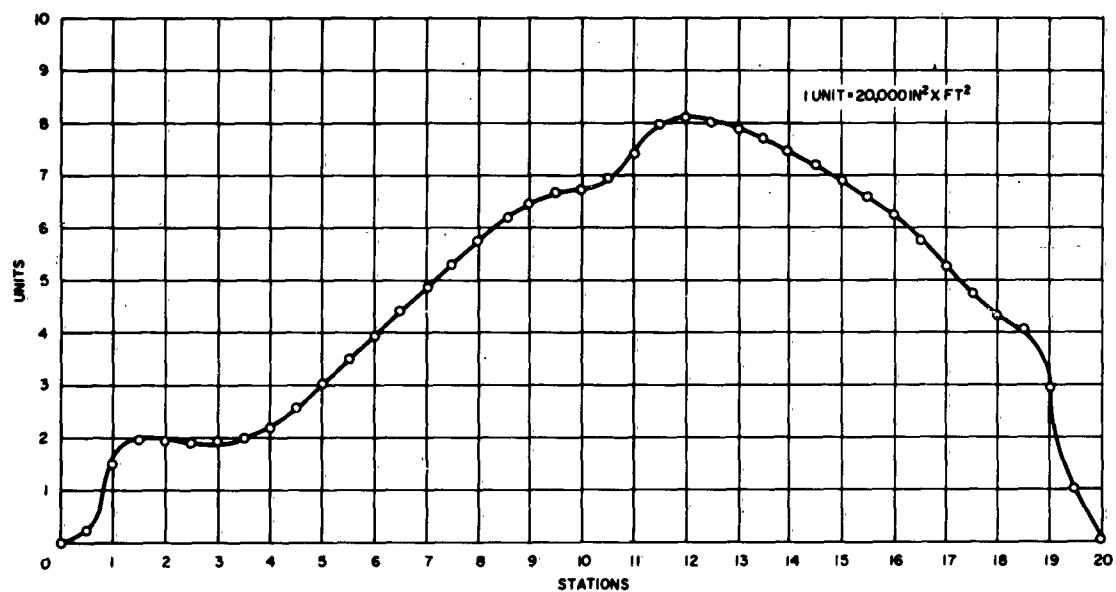


Figure 2 - Moments of Inertia Distribution for AGS 26 with respect to Horizontal Axis through Centroids of Sections

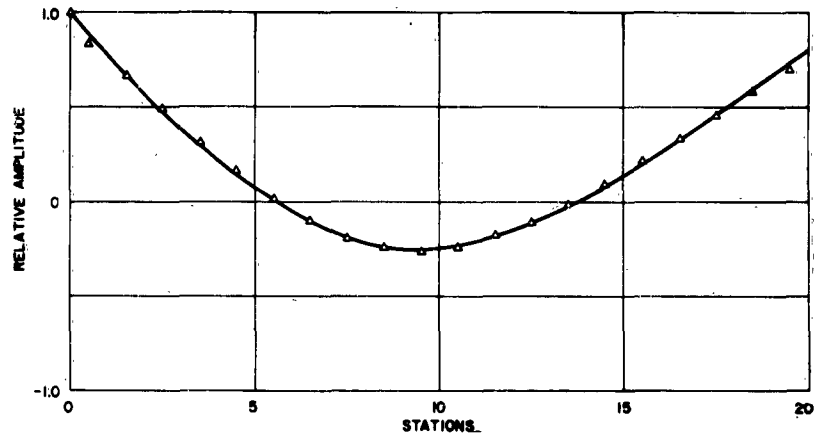


Figure 3a - 2-Noded Mode, 2.98 CPS

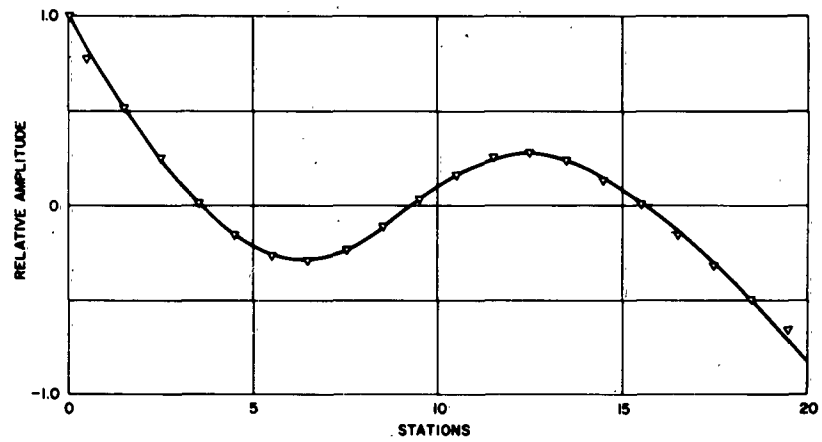


Figure 3b - 3-Noded Mode, 5.93 CPS

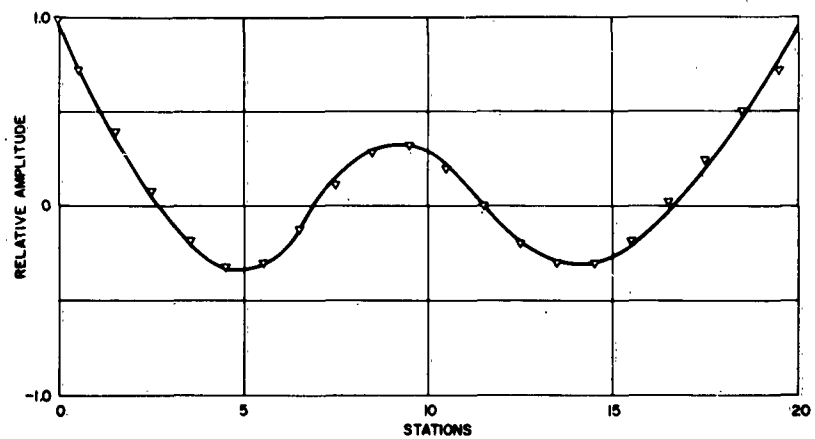


Figure 3c - 4-Noded Mode, 9.05 CPS

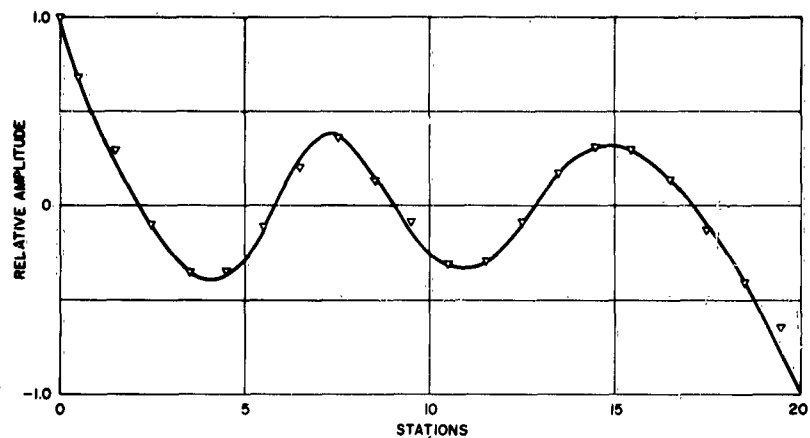


Figure 3d - 5-Noded Mode, 12.30 CPS

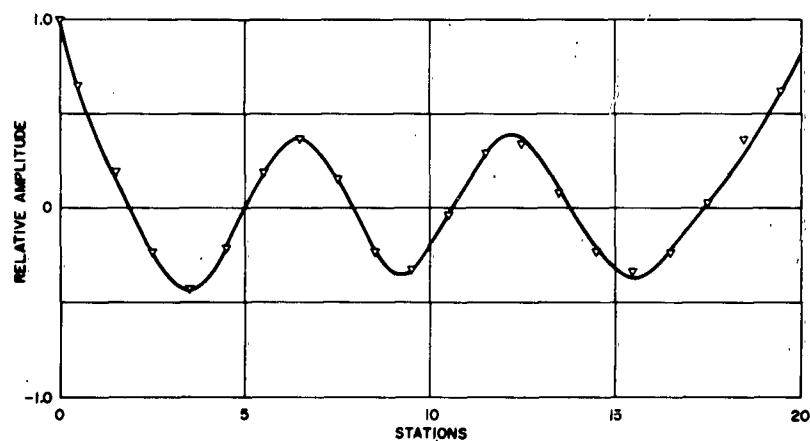


Figure 3e - 6-Noded Mode, 15.59 CPS

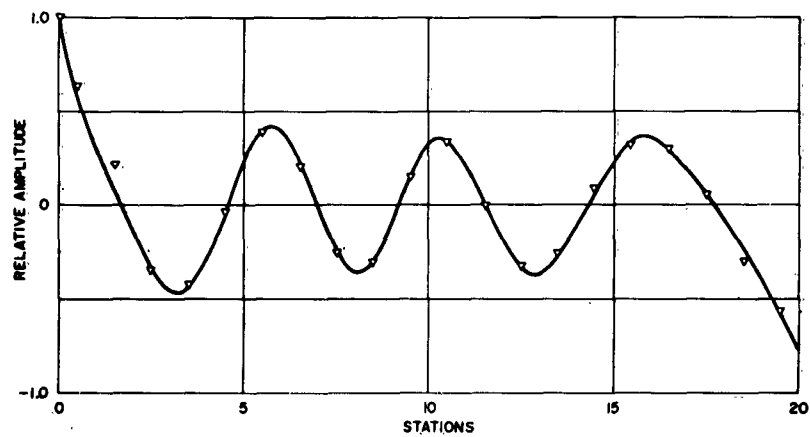


Figure 3f - 7-Noded Mode, 18.31 CPS

Figure 3 - Calculated Normal Mode Profiles and Natural Frequencies of Vertical Vibration

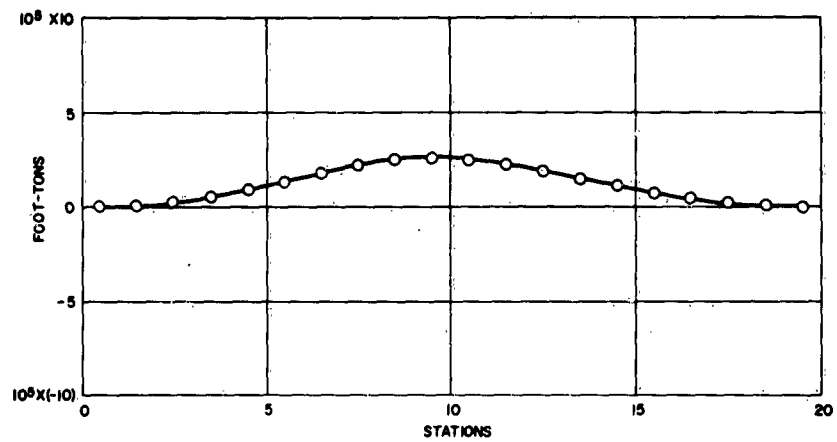


Figure 4a - 2-Noded Mode

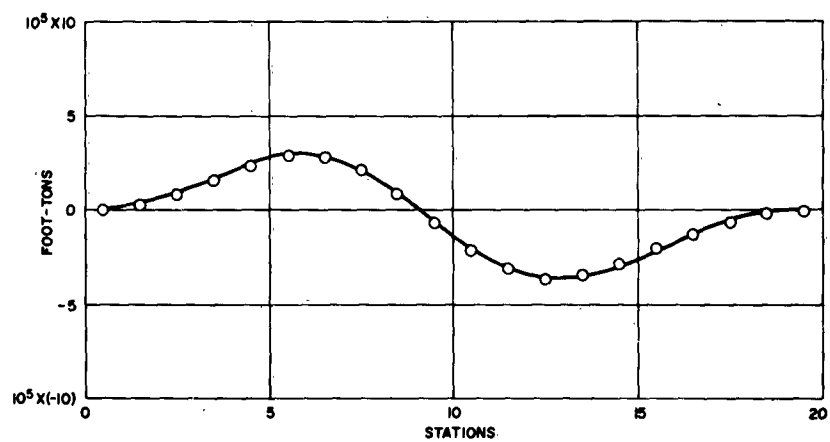


Figure 4b - 3-Noded Mode

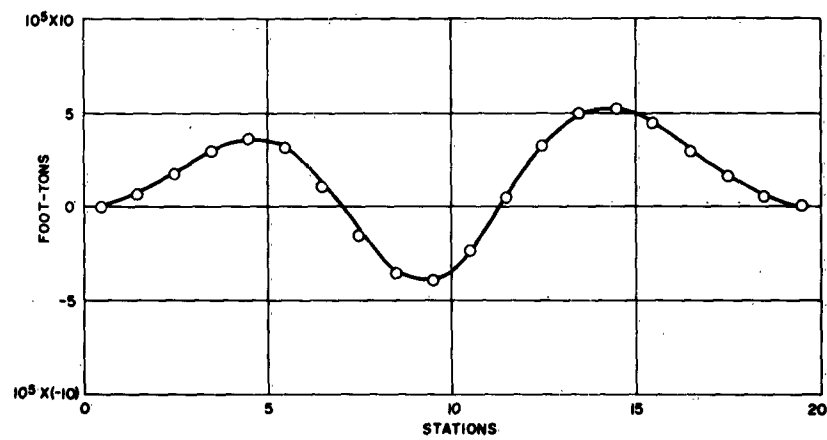


Figure 4c - 4-Noded Mode

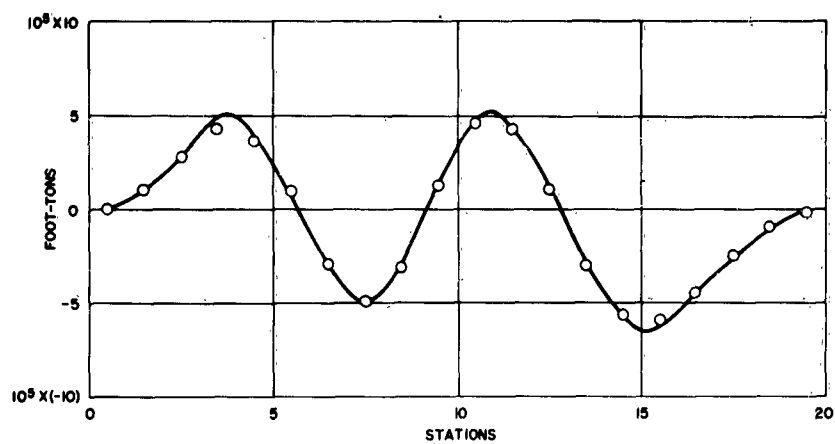


Figure 4d - 5-Noded Mode

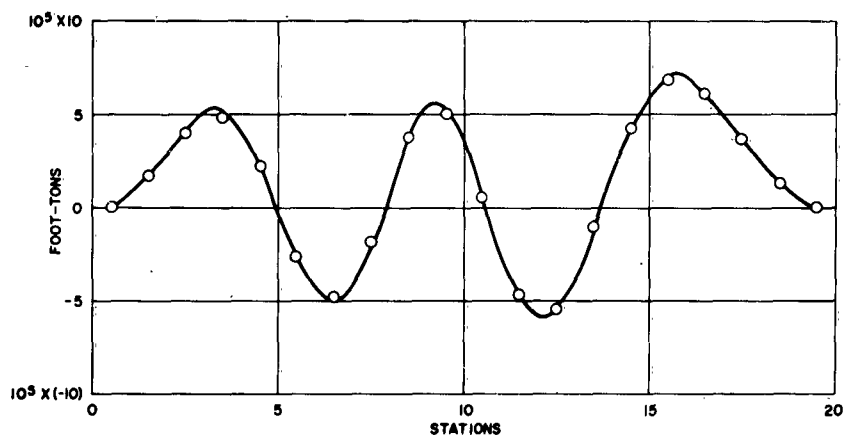


Figure 4e - 6-Noded Mode

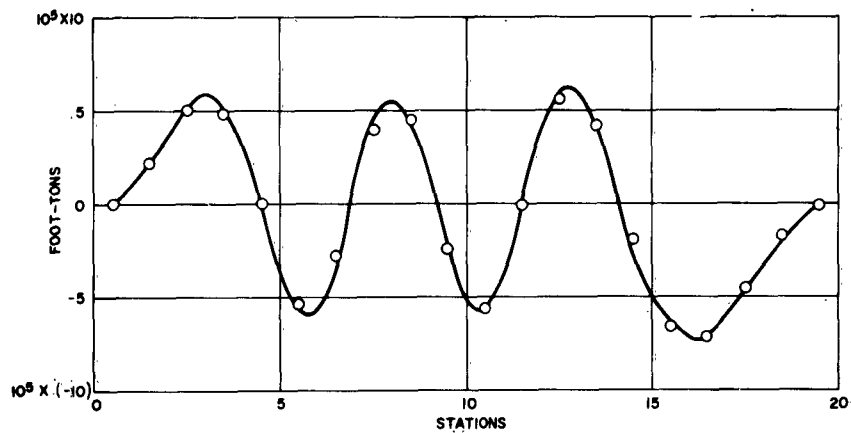


Figure 4f - 7-Noded Mode

Figure 4 - Moment Distribution for Vertical Flexural Vibration, Assuming Unit Deflection at Stern

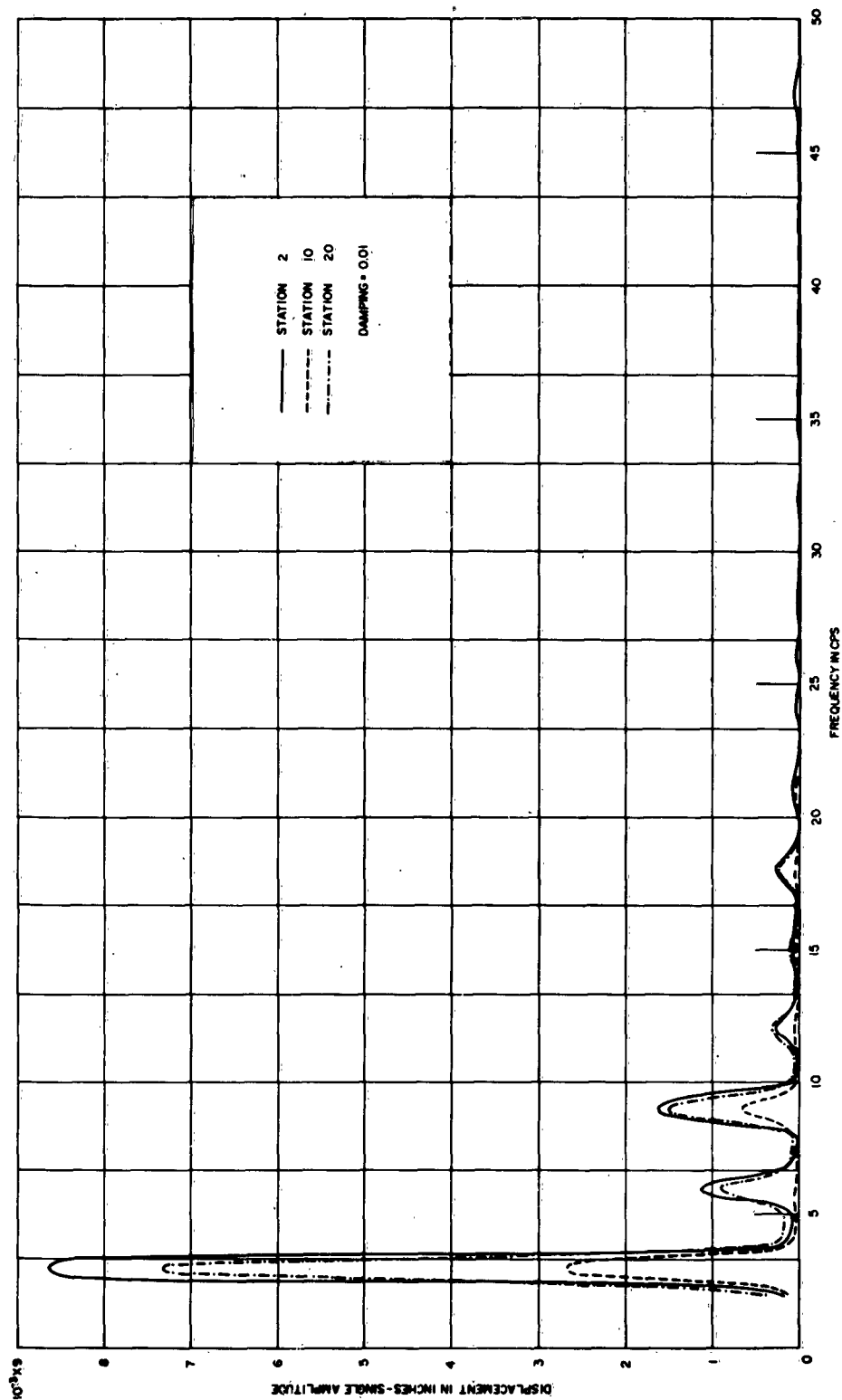


Figure 5 - Response to a 1-Ton Single Amplitude Sinusoidal Forcing Function at Selected Hull Stations, Damping Constant = 0.01 $\mu\omega$



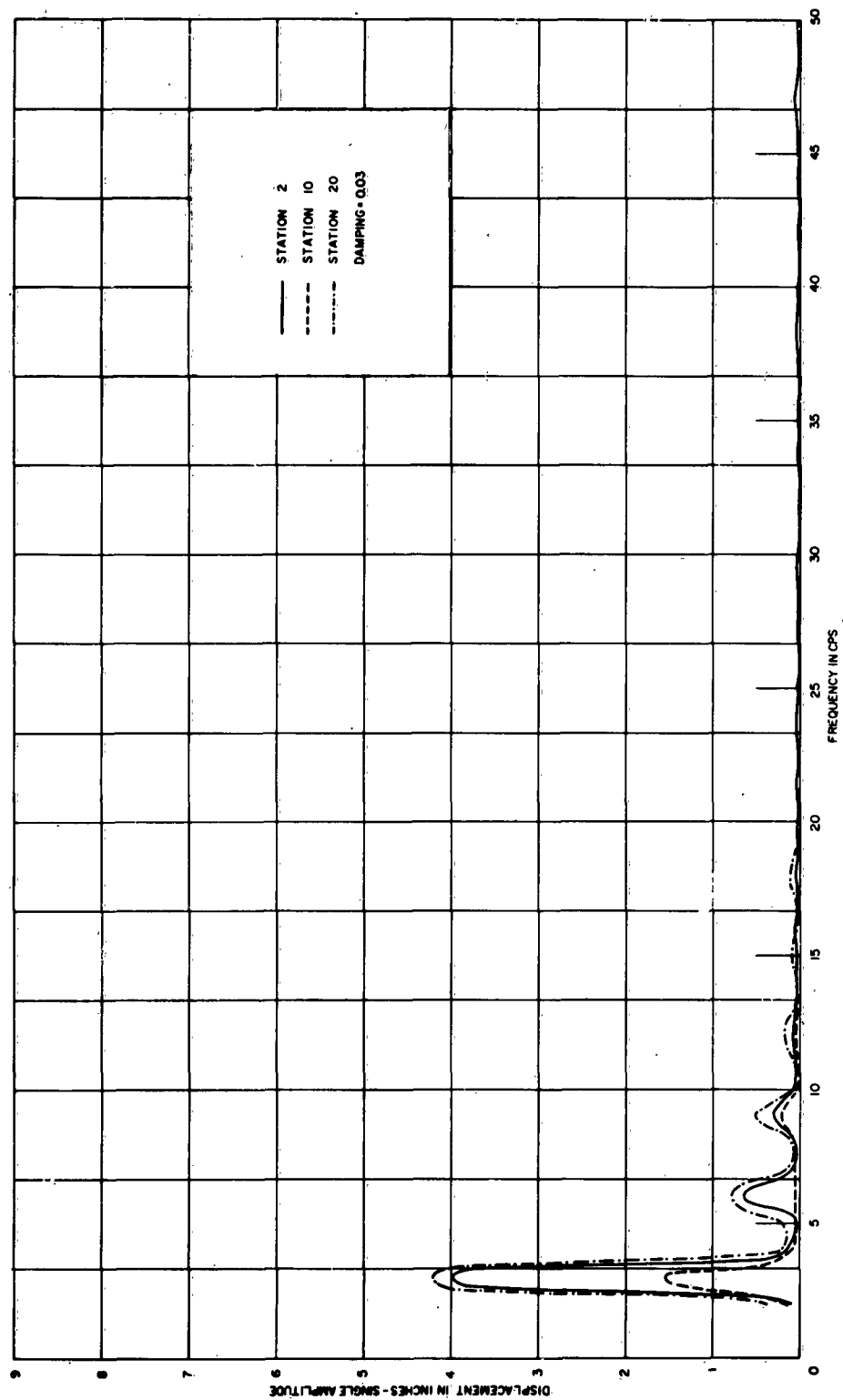


Figure 6 - Response to a 1-Ton Single Amplitude Sinusoidal Forcing Function at Selected Hull Stations, Damping Constant =  $0.03\mu\omega$

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